Phase Difference and Coherence as Diagnostics of Accreting Compact Sources

Xin-Min Hua[1], Demosthenes Kazanas and Lev Titarchuk[2]

LHEA, NASA/GSFC Code 661, Greenbelt, MD 20771

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[1]NRC/NAS Senior Research Associate

[2]CSI, George Mason University

ABSTRACT

We present calculations of the time lags and the coherence function of X-ray photons for a novel model of radiation emission from accretion powered, high-energy sources. Our model involves only Comptonization of soft photons injected near the compact object in an extended but non-uniform atmosphere around the compact object. Our results show that this model produces time lags between the hard and soft bands of the X-ray spectrum which increase with Fourier period, in agreement with recent observations; it also produces a coherence function equal to one over a wide range of frequencies if the system parameters do not have significant changes, also in agreement with the limited existing observations. We explore various conditions that could affect coherence functions. We indicate that measurements of these statistical quantities could provide diagnostics of the radial structure of the density of this class of sources.

Subject headings: accretion— black hole physics— radiation mechanisms: thermal— stars: neutron— X-rays

1. Introduction

It is believed that the bright galactic X-ray sources are compact objects (black holes and neutron stars) powered by thermalizing the accretion kinetic energy on the surface of the neutron star or near the black hole horizon. The X-ray emission is, then, naturally accounted for as the result of Comptonization of soft photons by hot electrons which are expected to be present in the deep gravitational potential of a compact object. In fact, the spectra of these sources have been modeled successfully with this process, which has been analyzed in great depth theoretically (see e.g. Sunyaev & Titarchuk 1980; Titarchuk 1994; Hua & Titarchuk 1995) and much information about the conditions of the accreting gas can be derived from fitting the high energy spectra to observations.

It is well known, however, that the Comptonization spectra alone cannot provide any clues about the dynamics of accretion of the hot gas onto the compact object. One needs, in addition, time variability information. However, since it is tacitly accepted that the X-ray emission originates at the smallest radii of the accreting flow, one expects that such information should reflect the dynamical time scales or the electron scattering time scales associated with that region. In fact, the observed energy spectra indicate Thomson depths of a few and thus guarantee these two time scales to be of roughly the same order of magnitude. The recent RXTE observations (W. Focke, personal communication; also Meekins et al. 1984) which resolve shots of duration $\sim$ msec, appear to provide a validation for this simplest expectations.

With these comments in mind, it appears strange that the X-ray fluctuation power spectral densities (PSD) of accreting compact sources contain most of their power at frequencies $\omega \lesssim 1$ Hz, far removed from the kHz frequencies expected on the basis of the arguments given above. This fact hints that one may have to modify the notion that the entire X-ray emission in this class of sources derives from a region a few Schwarzschild radii

in size. Alternatively, the lack of high frequency power in the PSD could be attributed to the much longer viscous times scales of accretion disks near compact objects, or to an overall modulation of the accretion rate with an (otherwise undetermined) power spectrum similar to the observed PSD.

While the above arguments could provide reasoning for the observed PSD form, they have hard time addressing observations associated with more involved tests of Comptonization induced variability and in particular the correlated variability at different energy bands. More specifically, Miyamoto et al. (1988, 1991) studied the time lags between the soft and hard photons in the X-ray light curves of Cyg X-1, using the GINGA data. It was shown in these references that the hard time lags increase roughly linearly with the Fourier period P from $P \lesssim 0.1$ sec to $P \sim 10$ sec, being of order $0.01P$ across this range of P. These long lags are very hard to understand in a model where the X-ray emission is due to soft photon Comptonization in the vicinity of the compact object. In such a model, they should simply reflect the photon scattering time in the specific region ($\simeq$ msec). The authors found the lag dependence on the Fourier period disturbing enough to question whether the process of Comptonization is indeed the one responsible for the formation of the X-ray spectrum of Cyg X-1. In fact, this dependence also rules out scattering in an extended, uniform, very low density X-ray corona, often invoked in models of accreting sources, since this too would produce time lags independent of the Fourier period.

More recently, Nowak & Vaughan (1996) and Vaughan & Nowak (1997, hereafter VN97) have brought attention to another statistic of importance in understanding the spatio-temporal structure of accreting sources, namely the coherence of the X-ray light curves. This is to some extent the normalized cross-correlation function of the light curves at two different energy bands obtained from an ensemble of measurements. These authors computed the coherence function for the GINGA data of Cyg X-1 and GX 339-4 and found

it to be equal to one for both sources over the frequency range 0.1 - 10 Hz. The coherence function for Cyg X-1 has also been computed with the more recent, higher quality data of RXTE (Cui et al. 1997b) and it was found to be equal to one up to frequency $\simeq$ 20 Hz. They consider this fact to be quite surprising, since most of the models they produce have coherence functions substantially smaller than those obtained from observations.

Motivated by the discrepancy between the expected and the observed variability behavior of accreting compact sources, Kazanas et al. (1997, hereafter KHT) proposed that the Comptonization process responsible for the formation of the spectra of accreting sources takes place in an non-uniform "atmosphere" which extends over several decades in radius. It was shown that this model can account for the form of the observed PSDs, the energy spectra and at the same time predicts a correlation between the slopes of the PSD and the energy spectra.

In the present paper, we test this model further by studying its hard X-ray time lags and coherence function and examine under what conditions agreement with observation can be obtained. In §2 we briefly review the model of KHT and then compute the associated time lags as a function of Fourier frequency. In §3 the coherence function is computed for the same model, while in §4 the results are reviewed and conclusions are drawn concerning possibility of uncovering the structure of accreting sources from spatio-temporal measurements.

2. Time and Phase Differences

The model of KHT considers Comptonization in a cloud of constant temperature but non-uniform density of profile $n(r) \propto r^{-1}$, which extends over several decades in the spherical radial coordinate from the compact source, r. In order to explore the

Comptonization in clouds with density configurations such as these, we developed a Monte Carlo method which can treat photon propagation and Compton scattering in inhomogeneous media. The method was described in detail by Hua (1997) and its first calculations were displayed in KHT.

The parameters of the calculations carried out in the present study were so chosen as to provide qualitative agreement of the resulting spectra with those of Cyg X-1 recently obtained by BATSE aboard CGRO. The CGRO/BATSE data, given by Ling et al. (1997), show that Cyg X-1 at its soft (γ_0) state has a spectrum consistent with Comptonization in an electron cloud of temperature ~ 110 keV and Thomson optical depth ~ 0.45. With this in mind, we employ a spherical model similar to that described in KHT but with temperature $kT_e = 100$ keV, in order to account for the excess emission at $\gtrsim 80$ keV. In fact, this spectrum extrapolates at low energies into the spectrum of Cyg X-1 obtained by RXTE (Cui et al. 1997a) at a different epoch but with the source in a similar spectral state. The entire source has a radius $r_2 \approx 1.5$ light seconds and consists of a central core of radius r_1 and an extended "atmosphere" with density profile $n(r) = n_1 r_1/r$ for $r_1 < r < r_2$. For the region $r < r_1$ we assume that it has a uniform density $n = n_1$, and that a soft photon source of blackbody spectrum at temperature 0.2 keV (Cui et al. 1997a) is located at its center (note that the density jump at $r = r_1$ used in KHT is absent in the present profile).

As was shown in KHT, the density gradient of the extended atmosphere can significantly affect the resulting photon spectrum relative to that resulting from a uniform configuration of the same temperature and total Thomson depth. However, with a redefinition of the total Thomson depth, to account for the more efficient photon escape in this non-uniform configuration in comparison with the uniform one, the resulting spectra can be made similar. Thus, the total optical depth of the source in our investigation should be larger than the uniform source value ($\tau_0 = 0.45$) used in Ling et al. (1997) to fit the BATSE data. We found

that a total optical depth $\tau_0 = 1$ for our non-uniform configuration produces as good a fit to the BATSE data as the one used by Ling et al. (1997). Furthermore, we assume three values for the radius of the central core of the cloud: $r_1 = 2.4 \times 10^{-2},\ 2.4 \times 10^{-3},\ 2.4 \times 10^{-4}$ light seconds. These conditions suffice to determine the density n_1, which is given by

$$n_1 = \frac{\tau_0}{r_1 \sigma_T \left[1 + \ln\left(\frac{r_2}{r_1}\right)\right]} \quad . \tag{1}$$

With the above configuration, we have calculated, using the Monte Carlo code, the energies and arriving times of the photons emerging from the cloud under consideration to a distant observer. The photons are collected in the energy bands $2 - 6.5$ and $13.1 - 60$ keV in order to simulate precisely the recent RXTE observation (Cui et al. 1997b). In time, the photons are collected in 4096 bins, each with a width 6/4096 seconds. Based on the light curves so obtained, we calculated the phase and time lags of the higher ($13.1 - 60$ keV) energy band with respect to lower one ($2 - 6.5$ keV) for clouds with the three different values of r_1 given above. The resulting time lags (solid curves) as well as the corresponding phase lags (dotted curves) as a function of the Fourier frequency are shown in Figure 1. It is apparent that these quantities have quite different form from those associated with Comptonization in uniform electron clouds (Hua & Titarchuk 1996). For the latter, the phase lag functions have maxima at a characteristic frequency roughly equal to the reciprocal mean escape time of the photons from the cloud. The phase lags obtained from the present model, however, are almost constant, extending from a low frequency of $\simeq 0.1$ Hz, characteristic of the electron scattering time at the largest, least dense part of the atmosphere, to a cut off frequency at ~ 100 Hz, determined by the time resolution of our calculations. Consequently, the corresponding time lags are power-laws of indices $\lesssim 1$, being at the lowest frequencies, as large as $\gtrsim 0.1$ seconds, in rough agreement with observations of Cui et al. (1997b) and Miyamoto et al. (1988, 1991) (the slight increase in the lags at

high frequencies is due to the effects of scattering in the uniform part of the configuration at $r < r_1$).

It is apparent, that the relation of the time lags to the Fourier frequency obtained in these simulations depends on the specific form of the radial dependence of density of the extended "atmosphere". To indicate the effect of such a dependence, we also display in the same figure, the phase and time lags resulting from an atmosphere with density profile $n(r) \propto r^{-3/2}$, extending over a similar range in r. The difference between the curves corresponding to the different density profiles is evident. The lags associated with the steeper profile have a lot weaker dependence on the Fourier frequency, because most of the photon scatterings, which give rise to the lags, take place near the radius at which the Thomson depth is highest, i.e., at the smallest radii. Once the photon leaves that region, it suffers very little additional scatterings and hence little additional lag occurs at lower Fourier frequencies. In this sense, the density profile used in KHT and in the present study, is special in obtaining agreement with observation, a fact whose importance has not escaped the authors. Similarly, for density profiles flatter than that used herein, which cuts-off beyond a certain radius to ensure finite Thomson depth, most of the lags are produced by scattering at the largest radii and would therefore be representative of the scattering time at that radius. It becomes apparent therefore, that the observations of frequency dependent hard X-ray lags not only argues in favor of the presence of the extended atmosphere used herein, but also point to their study as a means for probing its detailed density profile. Furthermore, the range of the Fourier frequencies over which the phase lags are constant is indicative of the range in radii over which the specific power law profile of the atmosphere extends. We plan to provide a more extensive investigation of these issues in future publications.

3. The Coherence Functions

In search of more probing tests of the variability of accreting sources, VN97 have examined the coherence function. It basically indicates to what extent the light curves at two different energy bands track each other *linearly* during the period over which the observations are made. These authors indicated that while most models of accreting compact sources have not been tested against this diagnostic, they generally tend to produce incoherent sources (Nowak 1994). Their conclusion was that it is very easy to destroy coherence and difficult to produce. Nevertheless, when analyzing GINGA data associated with Cyg X-1 and GX 339-4 they found (VN97) that the coherence function of these sources to be equal to one for Fourier frequencies below 10 Hz!

Motivated by these considerations we computed the coherence functions for the model outlined in the previous section. In order to examine what conditions would lead to loss of coherence in our model, we assumed that the configuration our system changes during an observation. To simulate this evolution, we used the light curves in the energy bands $2-6.5$ and $13.1-60$ keV resulting from two configurations represented by different parameters of our model. In order to estimate the noise due to the statistical nature of the Monte Carlo calculations, we produced a group of eight light curves for each energy band of each configuration, obtained by following the history of 10^6 photons with different initial random number seeds. The difference between each light curve and the average of the eight curves in the same group is taken as the noise. For the two energy bands given above, labeled 1 and 2 respectively, we used the 16 pairs of light curves, eight from each of two distinct configurations, and their respective noises to compute the coherence function defined in Equation (2) of VN97. The power spectra $|S_i|^2$ in this equation should be understood as being noise-corrected, that is, $|S_i|^2 = P_i - |N_i|^2 (i = 1, 2)$, where P_i and $|N_i|^2$ are power spectra obtained from the calculated light curve and the corresponding noise respectively.

(We are indebted to B. Vaughan for his insistence on this point.) The averages in the equation are taken over the 16 pairs of “measurements”.

For the purpose of verifying the above procedure, we first computed the coherence function of our model by averaging over 16 pair light curves from two identical configurations. We found that the coherence function of our model was 1 across the entire frequency range, as expected. We then computed coherence functions for a variety of configuration evolutions with the results presented in Figure 2. It is seen that changes in the parameters of our model do result in loss of coherence over the frequency range $1/6 - 2048/6$ Hz. The four thick curves represent the evolution of the configuration starting from one of those described in the above section, namely that with $r_1 = 2.4 \times 10^{-3}$ light seconds to one of the following configurations: (a) One with n_1 and τ_0 increased by a factor of 5 with the density profile unchanged (solid curve). In this case, the physical sizes r_1 and r_2 remain the same and the coherence reduces to < 0.6 over the entire frequency range with large statistical fluctuations at high frequencies. (b) One with electron temperature decreased from the initial value of 100 keV to 50 keV (dotted curve). In this case, the coherence is ≈ 0.85 below ~ 100 Hz and drops at higher frequencies. (c) Configurations with different energy of initial soft photons. While in the initial configuration the source photons have a blackbody distribution at temperature $kT_0 = 0.2$ keV, the dashed curve is obtained between this initial configuration and one with source photons of a single energy $E_0 = 13.1$ keV. It is seen in this case the coherence is reduced virtually to zero. On the other hand, if the final configuration has source photons at the blackbody temperature $kT_0 = 4$ keV, the coherence becomes ≥ 0.8 over the entire frequency range, as the dash-dotted curve indicates. (It has coherence slightly greater than 1 at high frequencies, probably due to noise overcorrection.)

Special attention should be paid to the thin curve in Figure 2. This represents an evolution between two configurations described in the last section, namely from that with

$r_1 = 2.4 \times 10^{-2}$ light second to that with $r_1 = 2.4 \times 10^{-4}$ light second (or vice versa). It is seen that the coherence is virtually unity over the entire range of frequencies under consideration, although the size of the uniform part of the configuration changes by two orders of magnitude. The cause of this outcome becomes apparent by taking a closer look to the PSD and phase lag of these two configurations. Since the light curves are obtained from two different configurations, they essentially represent two independent time series, say q and r for the two energy bands 1 and 2. One can then use Equation (10) in VN97 to understand their coherence. Using the same notation, we rewrite the equation in terms of the ratios of PSD in the two energy bands $\alpha_1 = |Q_1|/|R_1|$ and $\alpha_2 = |Q_2|/|R_2|$.

$$\gamma_I^2 = \frac{1 + \alpha_1^2\alpha_2^2 + 2\alpha_1\alpha_2\cos(\delta\theta_r - \delta\theta_q)}{(1+\alpha_1^2)(1+\alpha_2^2)}. \tag{2}$$

It is found that at low frequencies, the PSD in the two energy bands are in proportion $\alpha_1 \approx \alpha_2 \approx 1$. And from Figure 1, we see that the difference in the phase lag between these energy bands is small $\delta\theta_r - \delta\theta_q \lesssim 10°$; equation (2) then suggests that the coherence should be $\gamma_I^2 \simeq 1$. At higher frequencies, however, we found $\alpha_1 \neq \alpha_2$ and $\delta\theta_r - \delta\theta_q$ could be as great as 50° (see Figure 1). We might then expect the coherence to be less than one at these frequencies. But, in this frequency range, the values of α_1 and α_2 are found to be much smaller than 1, yielding $\gamma_I^2 \simeq 1$. Therefore the coherence at high frequencies is only superficial, since neither of the conditions outlined in VN97 are satisfied, and it is caused by the small values of α_1 and α_2 compared to one. In a similar way, one can examine the true causes for the coherences or lack thereof as displayed in Figure 2. For example, when we examine the case corresponding to the dashed curve in Figure 2, it was found that the near zero coherence over the entire frequency range is due to the fact that $\alpha_2 \ll \alpha_1$, so that $\gamma_I^2 \approx 1/\alpha_1^2 \approx 0.01$.

In view of these results, one should be cautious drawing strong conclusions from

measurements of coherence close to one, such as those of Cyg X-1 and GX339-4 given in VN97. That may indicate the constant state of the responsible mechanism over the observation time. On the other hand, it may also indicate changes in the system to which the coherence statistic is insensitive.

4. Discussion and Conclusions

We have presented above two statistics associated with a model for the time variability of the X-ray radiation emitted by accreting compact objects. In particular: Using a Monte Carlo code, we have computed the time lags between soft and hard photons and also the coherence of the X-ray light curves between the same energy bands resulting from Compton scattering in constant temperature but non-uniform density “atmosphere” which extends over several decades in radius. In our simulations we have assumed that the sole source of phase differences is due to Compton scattering of soft photons by the hot electrons of the atmosphere; the sole source of loss of coherence is the change of the system during observation. In addition, we assumed that the source of photons resides near the center of the configuration.

Our results indicate that:

1. The hard photons lag behind the soft ones by amounts which increase with the variability Fourier period. This behavior is distinctly different from that of Compton scattering by an electron cloud of uniform density and in agreement with observations in both the GINGA and the RXTE data.

2. The hard photon phase and time lags as a function of the Fourier frequency depend on the density profile of the extended scattering atmosphere. The observed lags are consistent with a density profile $n(r) \propto r^{-1}$ used in this study, with the range of frequencies

over which the phase lags remain constant reflecting the range of radii over which this density profile holds. Accurate measurements of the lag dependence on frequency could be used in the deconvolution of the density profile of the atmosphere from observations.

3. Our model generally produces coherence close to one. This, in view of the results of VN97 on the coherence of Cyg X-1, would indicate that the parameters of this system remain constant over very long time scales (hours). The coherence function between two energy bands, within our model, can be reduced to less than one by changing the macroscopic parameters of the Comptonization cloud and/or the energy of the source photons. However, the inverse of this statement is not true; coherence equal to one is, for practical purposes, only a sufficient condition for the parameters of the configuration remaining constant.

We believe that the simplicity and the ability of our model to correctly reproduce these variability statistics argues for the correctness of its basic premises, namely the presence of a non-uniform atmosphere and the effects of Compton scattering for determining the variability of accreting compact sources. We also believe that further scrutiny of this model by detail comparisons of all the information available, including the PSD and energy spectra, could determine the temperature and density structure of these accretion flows.

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FIGURE CAPTIONS

Fig. 1.— The time (solid curves) and phase (dotted curves) lags between energy bands $13.1-60$ and $2-6.5$ keV resulting from the extended atmosphere of temperature $kT_e = 100$ keV and optical depth $\tau_0 = 1$. Three cases of different sizes for the central core radius r_1 are shown. The electron density has the form $n_1 r_1/r$ for radius $r > r_1$ and n_1 for $r \leq r_1$. Also shown are the phase and time lags (dashed-dotted and dashed curves respectively) for a density profile $n(r) = n_1(r_1/r)^{3/2}$ with $r_1 = 2.4 \times 10^{-4}$ light seconds.

Fig. 2.— Various coherence functions between emissions in energy bands $2 - -6.5$ and $13.1 - 60$ keV from five pair of configurations of our model. (See text for the specific definition of each curve.)